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Reliability Improvement of a Solid Rocket Motor in Early Design Phases

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Designing a reliable system is difficult and costly, due to the design process to be followed and test requirements. It is important to consider reliability in very early design phases, and it is essential to select a baseline design with attainable high reliability. Therefore, in early phases, it is required to predict reliability and evaluate possible improvements for the reliability of the design alternatives in a fast manner to make necessary design decisions. In this study, a solid rocket motor is considered as a case study, and it is aimed to determine important parameters that affect the solid rocket motor reliability, to assess ballistic performance and casing structural reliability, and to find a new design point to evaluate possible improvement range for its reliability. The variations in dimensions and material properties, which are obtained from the previous data, are considered as the sources of failures and the limit states for acceleration, and the total impulse and the maximum stress in the casing are approximated by the response surface method. Monte Carlo simulation is used with the response surface functions to assess the failure probability and distributions of the rocket motor performance parameters. By considering the effect of the input parameters and distribution functions of the performance parameters, a new design point is proposed to decrease the total probability of failure.

Nomenclature

a_a	=	acceleration necessary to activate the fuse, g
a_L	=	acceleration of the rocket at launcher exit, g
$a_{\max}$	=	maximum acceleration of the rocket, g
$f_x(x)$	=	joint probability density function
$g(x)$	=	limit function for the performance
g_i	=	ith limit state function
n	=	number of random variables
P_f	=	probability of failure
P_{fi}^T	=	probability of failure for ith limit state function at a specific temperature T
St	=	tensile strength, MPa
T	=	temperature, °C
t	=	time, s
X_i	=	ith design variable
X_i^c	=	normalized value of ith design variable
μ_i^T	=	mean value for ith response at a specific temperature T
$\sigma_{\max}$	=	maximum stress at the casing
σ_i^T	=	standard deviation for ith response at a specific temperature T
Φ	=	cumulative distribution function of standard normal distribution
χ_i	=	limiting value for ith response

I. Introduction

TO BE competitive in the market, it is very important to design cost-effective and reliable products. For this purpose, it is necessary to consider reliability as an integral part of the design procedure. However, especially for the systems used in military applications, obtaining high-reliability values is generally difficult and costly, because it requires detailed analysis with long calculation times, repeated tests in different conditions, and the use of precise manufacturing processes.

Generally, at the early design phases, reliability is considered as a factor to evaluate design alternatives, and its weight on the decision is determined by evaluating the requirements of the new systems. However, reliability comparison is made by using historical data for similar systems, which are not always available. Therefore, the lack of data may lead to subjective decisions and unrealistic reliability goals. After making a choice among the alternatives and setting the reliability goals, reliability is generally included in the design by means of using tight tolerances and high safety factors, which (even though it increases the reliability) does not provide sufficient control over the reliability of the design and results in overly safe and expensive products.

In addition to the difficulty of including reliability in design, reliability tests are also difficult and they are important cost drivers in the life cycle of a system. To decrease the cost, it is necessary to evaluate all failure probabilities in the design. After decreasing these probabilities by preventive measures taken in the design, a reliability test should be performed to focus on verification of the reliability predictions, not to evaluate the failures.

When a rocket motor (which is a one-shot device) is considered, reliability assessment and reliability testing are very expensive and difficult, because the tests are destructive and test sample size is determined by the binomial law. For these reasons, reliability should be evaluated systematically and failure probability should be minimized as much as possible before the test stage. For that purpose, probabilistic methods can be used with some simplifications and considerable assumptions to evaluate reliability in early design phases.

Reliability analysis based on probabilistic methods is popular, because it gives reasonable estimates of probability of failure and can be used as a basis for the design optimization. This type of reliability analysis relies on statistical distributions applied to the input variables to assess reliability or probability of failure based on the output variable by specifying a limit, which is indeed a threshold

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value or a function beyond which failure is assumed to happen. Any response value passing beyond this limit is considered in the failure region [1]. The probability of failure is given by

$$P_f = P[g(x) \leq 0] = \int_{g(x)\leq 0} f_x(x) \cdot \mathrm{d}x \qquad (1)$$

Calculation of this integral analytically is very difficult in most cases. Therefore, approximations or numerical solutions are commonly used [2]. Some widely used methods to estimate the probability of failure are first- and second-order reliability methods [3–6] and Monte Carlo simulation (MCS) [7,8].

A. Monte Carlo Simulation in Reliability Calculations

In real life, many uncertainties affect the performance of a system. A complete design activity should include these uncertainties as random variables. In routine design activities, worst-case approaches are commonly used to handle uncertainties. However, in most of the cases, using a worst-case approach leads to too-expensive and too-safe designs. Moreover, for many systems, complex phenomena such as nonlinearities or parameter interactions hinder determination of the worst case without a comprehensive investigation of the effects of random variables that have known ranges but uncertain values for any particular time or event.

Monte Carlo simulation is a method that solves a problem by generating random numbers that obey certain rules and observing what fraction of those numbers obey some property or properties. The method is useful for obtaining numerical solutions to problems that are too complicated to solve analytically [9]. Monte Carlo simulation is a powerful tool to perform computer-based experiments to evaluate the system behavior under random effects. In Monte Carlo simulations, many different scenarios are evaluated sequentially by generating random values for all random variables by inversely calculating their cumulative distribution functions and analyzing the system performance with these randomly generated numbers. Monte Carlo simulation can be used for many different kinds of problems, as noted in [2]:

> To understand what kinds of problems are solvable by this method, it is important to note that the method enables simulation of any process for which the development is influenced by random factors. Monte Carlo simulation is a widely used technique for probabilistic structural analysis, serving two main purposes: 1) validating analytical methods and 2) solving large, complex systems when analytical approximations are not feasible.

Monte Carlo simulation is one of the most commonly used methods for reliability calculations. With an n-dimensional random-variable vector, it is difficult to calculate the joint probability integral given in Eq. (1) by using analytical reliability methods. Therefore, the Monte Carlo method is generally preferred to estimate probability of failure. The first step in performing Monte Carlo simulation to estimate reliability is defining relations between inputs and responses. Then random numbers are generated for the random variates according to their assumed probability distributions and the responses are evaluated with these numbers. If any of the calculated responses violates the limits, the considered random-number set is marked as a failure. These steps are repeated until a predetermined sample size is reached. The more iterations done, the lower the standard error of the mean result, because the error is inversely proportional to the square root of the iteration number [2].

B. Response Surface Method

The response surface method is a powerful design of experiment methodology that gives a rapid estimate of the local response of a system by a closed-form formulation, which consists of interactions and higher-order effects. To obtain a response surface, the system is simulated several times for different configurations and the response data of the system is collected. These data points are then used to fit a plane or hypersurface to the response. By using the response surface method, system behavior is estimated by a polynomial function that simplifies the Monte Carlo simulation procedure. This presents an efficient way to handle the reliability problem by decreasing the calculation time for a fixed number of iterations.

Because the reliability calculations are generally performed in a narrow range due to relatively small uncertainties and tight tolerances, a linear response is usually sufficient. However, for the systems with high nonlinearities, a first-order model would show a significant lack of fit and a higher-order model such as a cubic response surface function given in Eq. (2) must be used:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1,j=i+1}^{k} \beta_{ij} X_i X_j + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i=1}^{k} \beta_{ii} X_i^3 + \sum_{i=1,j=i+1,h=j+1}^{k} \beta_{ijh} X_i X_j X_h + \sum_{i=1,j=1,i\neq j}^{k} \beta_{ij} X_i^2 X_j \qquad (2)$$

The prediction ability of the response function is generally evaluated by the coefficient of determination, which is the square of the correlation coefficient of the sample responses and predicted values for these sample points. As the coefficient of determination is calculated with the same data used to construct the response function, in some cases, it may overestimate the prediction quality. Another effective way of evaluating prediction quality is the cross validation of the response function and the design tool used to obtain real responses. After fitting a surface to the data, new responses are evaluated by the design tool and, at the same time, by the fitted response function at the same randomly generated cross-validation points. These points are generated purely for cross-validation studies and are therefore not used for any other purposes such as healing the fits. Then expected values that are obtained from the design tool and predicted values obtained from the response surface functions are plotted against each other. Obtaining points on the $x = y$ line is accepted as a strong evidence of high prediction ability without overfitting. The more the scatter of the points on the graph resembles the $x = y$ line, the better the fit.

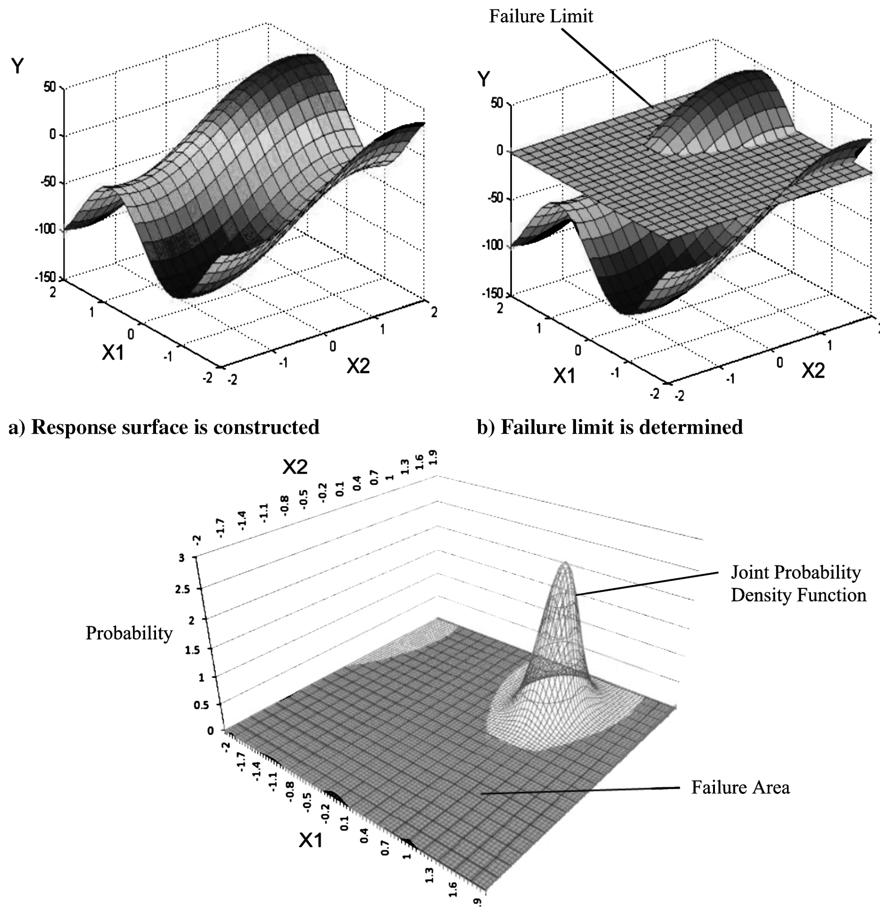
Although there are many types of design of experiments methods, one of the most common designs for response surface modeling is the central composite circumscribed (CCC) design. In this design, five levels of each variable are considered: 1 at the center, 2 at the limits (used to construct factorial design), and 2 beyond the limits of the variable (to form star points). CCC design can simply be formed by adding star points and a center point to a factorial design, resulting in a number of design points equal to $2^n + 2n + 1$. However, depending on the problem and time constraints, instead of using a full factorial design, a fractional factorial design that is indeed a special portion of the full factorial design can be used to build the CCC design to decrease the number of experiment runs.

II. Reliability Estimation Procedure

In this study, it is aimed to evaluate the solid rocket motor reliability with a simplified procedure and to make minor changes in the design to increase its reliability.

A procedure is described to improve the reliability without making major changes in the design so that the detailed design stages may start with a more reliable base design. All probability calculations are performed by using the Monte Carlo simulation and the functions used in the simulation are the approximations of the solid rocket motor performance responses obtained by the response surface method [10,11]. The experimental results, which are required to form the response surfaces, are obtained from the one-dimensional computer simulation of the ballistic performance and the structural analyses performed with ANSYS finite element software. The reliability prediction part of the procedure is summarized in Fig. 1 with two random variables and a generic response surface function that leads to failure when the response is smaller than zero.

Main steps of the procedure are as follows: First, the limit state functions (i.e., the performance of the system) and the input parameters affecting these limit states are determined. Second, the response functions that are valid in the variation range of the input parameters are formed by using the response surface method (RSM)



a) Response surface is constructed

b) Failure limit is determined

c) Joint probability density function is integrated over the failure area

Fig. 1 Illustration of the reliability estimation procedure.

and these functions are validated by the cross validation of the fitted function with the results of the ballistic performance prediction software or finite element analysis software. Then, with the response functions, the Monte Carlo simulation is performed to assess the current reliability of the system and variations of the responses.

After determining the current state of the system, sensitivity analysis is performed to explore the effects of the input parameters on the system performance and its variation. Then the input parameters, which are to be changed, are determined by considering their effects on the system. The new response surface functions that are valid in the optimization range are fitted to the limit states. In this step, because the variations in responses could be expressed with a known distribution function (normal distribution), the probability of failure due to each limit state is simply expressed as a normal distribution probability function and the calculations are simplified. Then, to find a new design point, the summation of these probability functions is minimized. In this step, it is assumed that failure associated with a limit state is purely uncorrelated with other limit states, and shapes of the distribution functions of each response remain constant. Finally, the Monte Carlo simulation is repeated for the new design point to validate the optimization results and to find a better estimation for reliability by considering the correlation among the limit state functions. This procedure is also summarized in Fig. 2.

Because reliability is defined as the probability of an item to perform its required functions under stated conditions, any case in which the rocket motor cannot perform its function can be considered as a failure. Although reliability of a rocket motor depends on its components' reliability and many limit states can be defined, depending on the failure mode of the rocket motor, failures related to the rocket motor casing and ballistic performance are taken into account in this study. Evaluating component failures would require data obtained in later design phases.

III. Limit State Functions

Performance requirements of the rocket motor studied are determined by considering the mission requirements of the rocket. These performance requirements are required to be fulfilled in a temperature range of -35 to $+60^\circ$C. The total impulse of the motor shall be greater than 7500 $\mathrm{N \cdot s}$ to obtain the desired range. Then the limit state function for this requirement is

$$g_1 = I_t - 7500 \tag{3}$$

To activate the warhead fuse of the rocket, the rocket motor must supply an acceleration greater then 27 g for at least 0.63 s:

$$g_2 = \Delta t_{(a_a > 27\ g)} - 0.63 \tag{4}$$

The acceleration of the rocket at launcher exit, a_l, shall be greater than 35 g, and the limit state function g_3 is

$$g_3 = a_L - 35\ g \tag{5}$$

As a loading limit obtained from the structural capacity of all components of the rocket motor, the maximum acceleration of rocket shall be less than 100 g:

$$g_4 = 100 - a_{\max} \tag{6}$$

In addition to the ballistic performance, the structural performance of the casing, which has a nominal thickness of 1.84 and is made of 2014-T6 aluminum alloy, is also estimated. The maximum stress in any part of the motor case shall be less than the tensile strength of the casing material:

$$g_5 = S_T - \sigma_{\max} \tag{7}$$

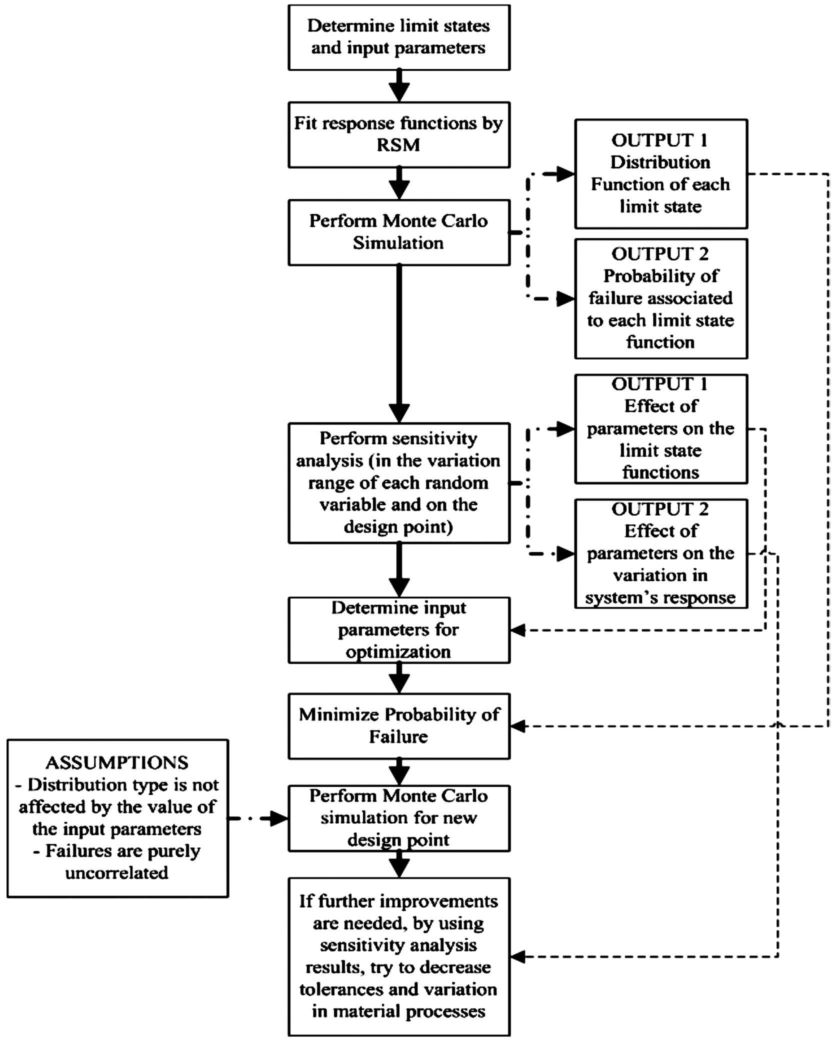


Fig. 2 Flowchart followed in the study.

IV. Design Variables Included in the Calculations

The ballistic performance parameters such as internal pressure, total impulse, and acceleration profile depend on the rocket propellant properties and motor dimensions. The parameters used to form a response surface for the ballistic performance are as follows:

1) Propellant properties are burn rate constant X_1, density X_2, enthalpy of the combustion gases X_3, and temperature sensitivity X_4.

2) Motor dimensions are tapered angle of the port X_5, grain length X_6, nozzle throat diameter X_7, nozzle exit diameter X_8, and propellant grain geometry parameters X_9 to X_{12} (Fig. 3).

In addition to the parameters given previously, the structural parameters are also defined for the casing structural reliability calculations. The random variables included in the response surface models to estimate structural reliability of the casing are as follows:

1) The load is the chamber pressure.

2) The dimensions are the case thickness X_{13}, case thickness at the nozzle interface X_{14}, and thickness of the lock wire at the nozzle joint X_{15}.

3) The case material tensile strength X_{16}.

The parameters X_1 to X_{15} are all considered as random variables and the coefficient of variation values, which are the ratios of the standard deviation to the mean value of each random variable, are given in Table 1. The coefficient of variation values for the material

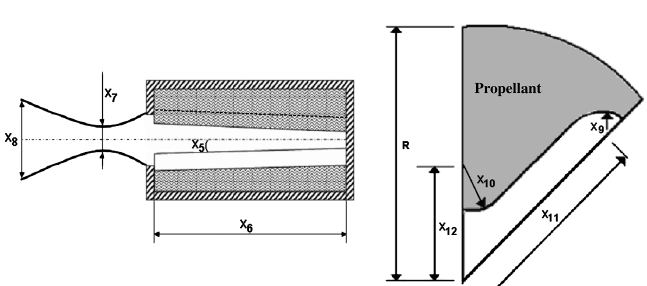


Fig. 3 Parameters describing the motor dimensions.

Table 1 Coefficient of variation values of random variables

Variables	Values
X_1	0.0283
X_2	0.0033
X_3	0.0088
X_4	0.35
X_5	0.029
X_6	0.0047
X_7	0.008
X_8	0.021
X_9	0.005
X_{10}	0.01
X_{11}	0.0007
X_{12}	0.0009
$X13$	0.023
$X14$	0.015
$X15$	0.004
$X16$	0.058

properties are obtained from the available experimental data, and the values for the dimensions are determined by considering the tolerances and process capability data.

In addition to these random variables, ambient temperature also has an effect on the ballistic performance, and therefore analyses are performed at different operating temperatures: −35, 20, and 60°C.

V. Validation of the Fitted Response Functions

The ballistic performance measures are estimated with a central composite circumscribed design with $1/32$ fraction. To evaluate the accuracy of the estimated functions, cross-validation studies are carried out. The results of the ballistic performance prediction software are compared with response functions at 100 random points for each limit state. The same procedure is applied to the response function fitted for the maximum stress in the casing. In this case, however, due to time-consuming finite element calculations, 10 random points instead of 100 are used to examine the accuracy of the fitted function. It is seen that expected values obtained from ballistic performance prediction software and finite element calculations are precisely predicted by response surface functions fitted. The validation results for different rocket performance parameters are shown in Figs. 4–8 for a +60°C ambient temperature.

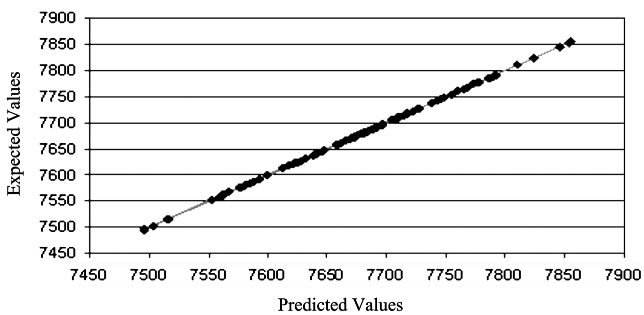


Fig. 4 Expected-vs-fitted values for total impulse, N · s (maximum absolute error is 1.36).

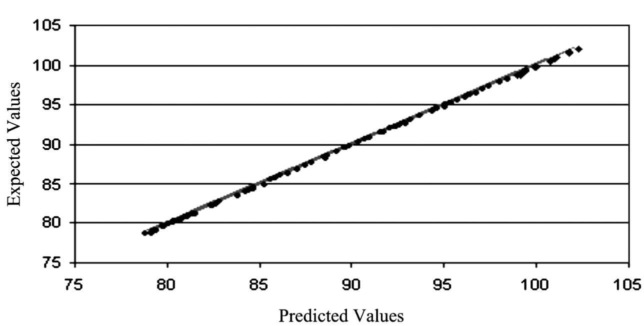


Fig. 5 Expected-vs-fitted values for maximum acceleration, *g* (maximum absolute error is 0.358).

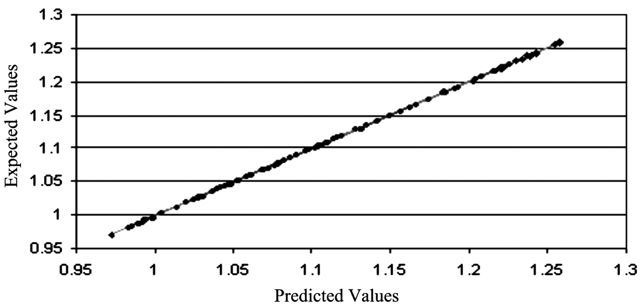


Fig. 6 Expected-vs-fitted values for arming acceleration duration, s (maximum absolute error is 0.0023).

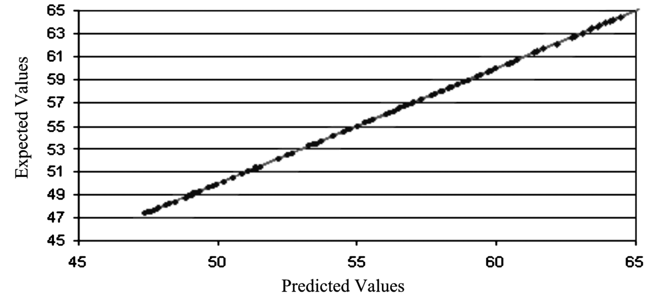


Fig. 7 Expected-vs-fitted values for launch acceleration, *g* (maximum absolute error is 0.113).

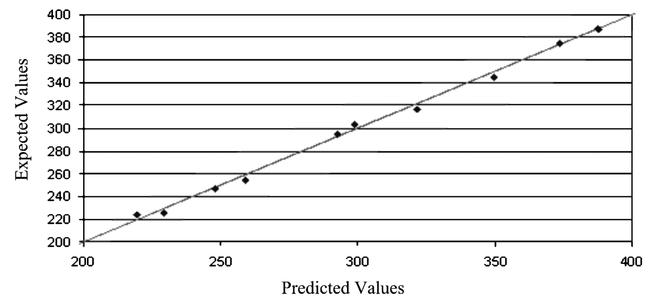


Fig. 8 Expected-vs-fitted values for maximum stress in the casing, mpa (maximum absolute error is 5.11).

VI. Estimated Distributions and Probability of Failure

After fitting and validating the response functions, the Monte Carlo simulation is performed to estimate variations in these performance parameters. In addition to these parameters, the mean and standard deviations of the maximum chamber pressure are also estimated. Maximum chamber pressure is actually the loading case in which maximum casing stress occurs and is therefore an input for the structural reliability calculation. All ballistic response parameters are found to be normally distributed (Fig. 9), and the estimated mean and standard deviations are given in Table 2. Results of the Monte Carlo simulation are also used to assess the probability of failure of the rocket motor due to the limit states g_1 to g_5 with a million iterations. The estimated probability of failure values is given in Table 3.

Considering results seen in Table 3, it can be said that exceeding the maximum acceleration limit at +60°C and obtaining an impulse lower than the limit at −35°C are the most probable failure modes for the ballistic performance.

VII. Determination of Attainable Reliability

In previous sections, the reliability of the rocket motor was estimated to be 0.988561, 0.998884, and 0.973227 for three different ambient temperatures: +60, +20, and −35°C, respectively. These values can be used for comparison purposes. However, without affecting the other design constraints of the system, these reliability values can be increased with limited effort. In this case, a changeable input parameter should be set to a new value, which does not require process improvements and does not affect the manufacturing cost. Moreover, new values should be selected near the current design point to avoid design changes on other components not considered in this study. For example, increasing the nozzle throat and exit diameters would decrease the nozzle material thickness, which may lead to structural failure of the nozzle. Therefore, to obtain meaningful results for the overall system, new design point is searched within the 10% range of each parameter. This is an engineering judgment made by considering the sensitivity analysis results and distances to the limit values. However, extending this study to include other components and functions would allow expansion of this range.

In many cases, dimensional parameters are easier to change than material properties. Therefore, parameters such as nozzle throat

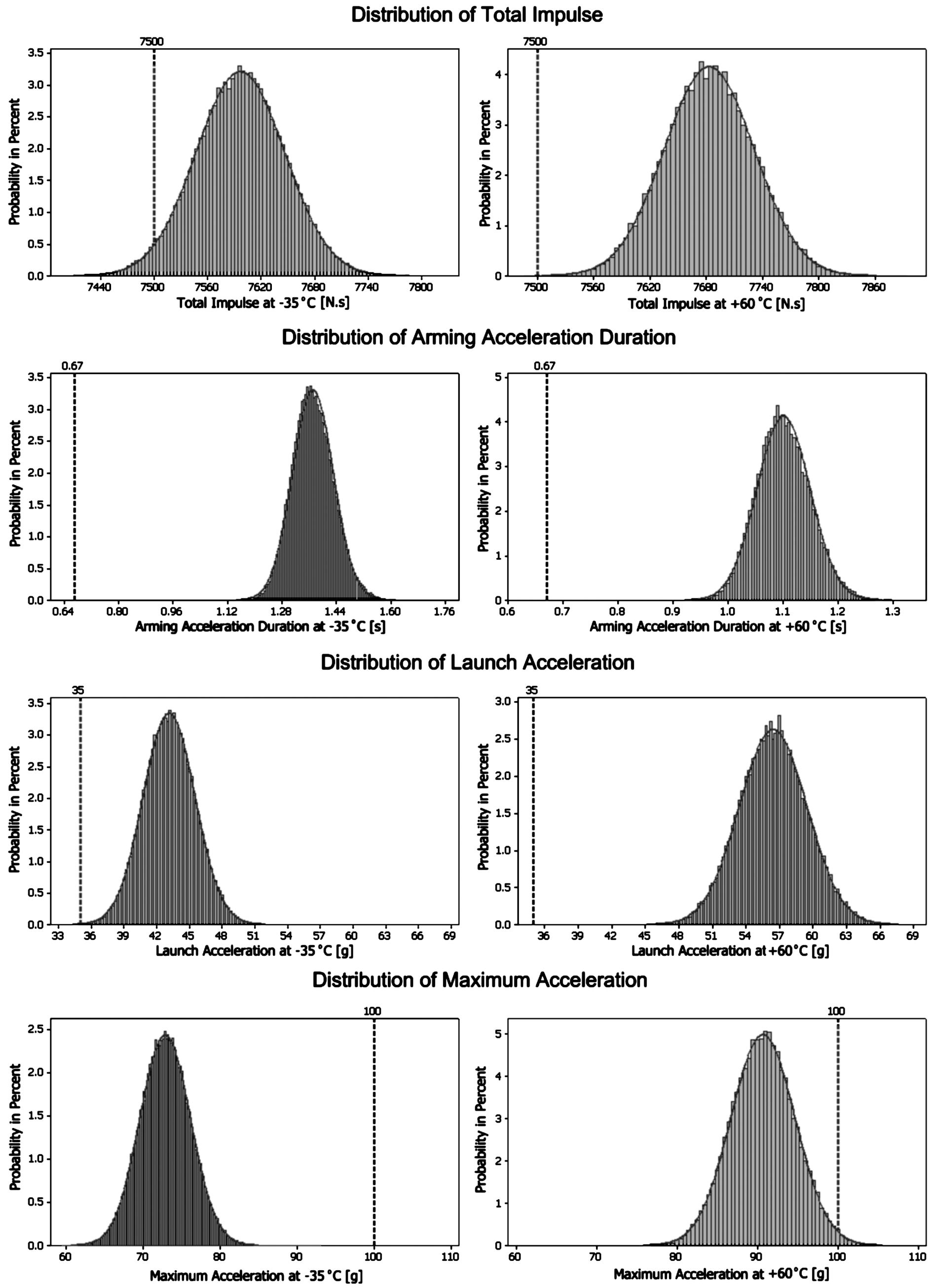


Fig. 9 Histogram plots for response parameters.

diameter, nozzle exit diameter, and grain geometry parameters should be preferred as the input parameters for the optimization problem. Additionally, the input parameters used in the optimization can be determined by using the sensitivity analysis results, in which the importance of each variable can be simply determined by evaluating the partial derivative of each response function in the direction of that particular random variable. For this purpose, the sensitivities of the response functions are calculated at the design point, and the parameters are listed in Table 4 with the order of importance for the rocket motor performance.

However, from the view of reliability, instead of using a point, sensitivity analysis should be performed by considering the variation range of the input parameters. In addition to the effect of input parameters on the rocket motor performance, their effect on the variation of rocket motor performance should also be calculated. Hence, the sensitivities of ballistic performance parameters are

Table 2 Variations in ballistic performance parameters

	−35°C			+60°C		
Performance parameter	Mean	Standard deviation	Distance to the failure limit σ	Mean	Standard deviation	Distance to the failure limit σ
Total impulse, N · s	7597	49.4	1.96	7683.7	48.2	3.81
Maximum acceleration, g	72.827	3.266	8.32	90.741	4.008	2.31
Arming acceleration duration, s	1.3713	0.0602	12.43	1.1007	0.0485	9.85
Launch acceleration, g	43.142	2.374	3.43	56.442	3.048	7.03
Maximum chamber pressure, bar	97.6	4.26	——	121.18	5.34	——

Table 3 Probability of failure and reliability at different operating temperatures V

	Estimated probability of failure		
Performance parameter	60°C	20°C	−35°C
Total impulse	0.000070	0.001109	0.026693
Maximum acceleration	0.011369	0.000007	$<10^{-6}$
Arming acceleration duration	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$
Launch acceleration	$<10^{-6}$	$<10^{-6}$	0.000211
Casing failure	0.000003	$<10^{-6}$	$<10^{-6}$
Total probability of failure	0.011439	0.001116	0.026773
Reliability	0.988561	0.998884	0.973227

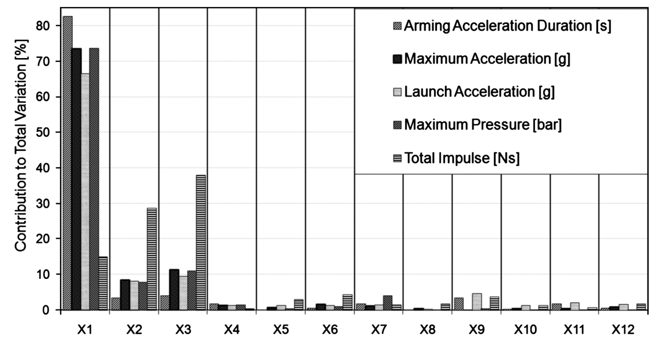


Fig. 10 Effect of each input parameter on the variation of the rocket motor performance.

recalculated by multiplying the response functions' slopes with the coefficient of variation associated with each variable. The sensitivity values of input parameters in the range of production tolerances are shown in Fig. 10, in terms of percent contributions to the variation of the rocket motor performance. The parameters that effect the response variations most should be preferred for a variation reduction study, which constitutes the last step of the flowchart given in Fig. 2.

Because they can easily be changed in the design, dimensional parameters are selected to be the input parameters for the optimization problem instead of material properties. The dimensions, which can be used in the optimization process, are selected to be the nozzle throat diameter, nozzle exit diameter, and grain geometry parameters X_9 to X_{12}.

From the sensitivity analysis, it is found that the grain geometry parameters R_1 and R_2 are not effective on the rocket motor performance. Therefore, a new design point with reduced probability of failure is searched by changing the nozzle throat diameter, nozzle exit diameter, and grain geometry parameters X_{11} and X_{12}.

After defining the input parameters and their ranges, the new response surfaces, which are valid in a 10% variation range of the input parameters, are fitted for two extreme temperatures: −35 and +60°C. These response surface functions are used to estimate the mean values of the response parameters: total impulse, maximum acceleration, launch acceleration, and arming acceleration duration. In addition to these response parameters, a response function is also fitted for the maximum chamber pressure, which is used to estimate the maximum stress at the rocket motor casing. The new response surface functions are fitted by using the circumscribed central composite design of experiments.

Because the objective is to minimize the total probability of failure, the probability of failure of the rocket motor must be expressed as an analytical function. As the rocket motor performance parameters are found to have normal distribution characteristics, it is assumed that the probability of failure associated with each limit state function can be calculated by using the cumulative distribution function of normal distribution. Another major assumption is that the standard deviation of performance parameters does not change when nozzle throat diameter, nozzle exit diameter, or grain geometry parameters X_{11} and X_{12} are changed. Furthermore, it is assumed that failure for each limit state function is purely uncorrelated with other failures. In reliability predictions by Monte Carlo simulation, when any of the requirements are not satisfied, it is considered as a failure. Therefore, by the nature of the process, correlations are added to the prediction. However, because correlations among the responses are neglected in the optimization procedure to simplify the problem, optimization results are affected as if an importance weight is used for the failure probabilities associated with responses with high correlation. Nonetheless, an improvement in overall reliability can be expected.

Although fitting a distribution to data may result in poor accuracy for low-density regions (i.e., tails of the normal distribution), with the distributions estimated so far, the probability of failure associated with each limit state can be calculated separately by using normal distribution probability function, because the new design point is also validated by Monte Carlo simulation after determining the new point analytically. Therefore, if there is a constant lower-limit value for the performance, the probability of failure for a limit state function at an ambient temperature T can be written as

$$P_{fi}^{T} = \Phi\left(\frac{\chi_i - \mu_i^T}{\sigma_i^T}\right) \tag{8}$$

If there is a constant upper limit for the performance, the same probability of failure can be written as

$$P_{fi}^{T} = 1 - \Phi\left(\frac{\chi_i - \mu_i^T}{\sigma_i^T}\right) \tag{9}$$

Then the optimization problem to determine the new dimensions can be defined as follows:

$$\text{Minimize} \quad \sum_{i=1}^{5}\sum_{j=1}^{2} P_{f_i}^{T_j} \qquad \text{subject to} \quad -1 \le X_7^c \le 1$$
$$-1 \le X_8^c \le 1 \qquad -1 \le X_{11}^c \le 1 \qquad -1 \le X_{12}^c \le 1 \tag{10}$$

where $T = [-35, 60]$. With the Newton search algorithm, the new

Table 4 Order of variables affecting rocket motor performance

Order of importance	Total impulse	Maximum acceleration	Arming acceleration duration	Launch acceleration	Maximum pressure
1	X_2	X_1	X_1	X_{11}	X_7
2	X_6	X_2	X_{11}	X_6	X_1
3	X_3	X_6	X_7	X_2	X_6
4	X_{12}	X_7	X_6	X_1	X_2
5	X_7	X_3	X_2	X_{12}	X_{12}
6	X_{11}	X_{12}	X_3	X_7	X_{11}
7	X_8	X_{11}	X_{12}	X_3	X_3
8	X_1	X_8	X_9	X_9	X_4
9	X_9	X_4	X_4	X_{10}	X_9
10	X_{10}	X_5	X_8	X_4	X_{10}
11	X_5	X_{10}	X_{10}	X_8	X_5
12	X_4	X_9	X_5	X_5	X_8

Table 5 Changes in the design parameters

Input parameter	Old value, m	New value, m	Old value (coded)	New value (coded)
Throat diameter	0.01175	0.01238	0	0.531915
Exit diameter	0.06168	0.06478	0	0.501822
Grain geometry parameter, X_{11}	0.0156	0.0152	0	−0.24863
Grain geometry parameter, X_{12}	0.0111	0.0102	0	−0.78078

values are estimated as given in Table 5. The predicted changes in the responses after setting new design point are given in Table 6.

Next, the estimated reliability of the rocket motor and distribution of the response parameters are recalculated by using the same procedure applied in the previous section. This way, reliability is estimated by considering the correlation between the failures, and validity of the optimization results are checked by comparing the expected mean values obtained in the optimization calculations and results obtained from the simulation (Table 7). The estimated probability of failure values at the new design point are listed in Table 8 for +60, +20, and −35°C.

By comparing the expected values after optimization and the simulation results, which are obtained by using the new design point, it can be said that the response function used in the optimization procedure gives acceptable results. Therefore, it can be concluded that the reliability of a system can be easily increased with small changes in the design parameters during the early design phases without any additional cost. However, if the target reliability cannot be reached after changing the dimensions to an optimum point, then improvements in manufacturing processes and the material properties should be considered. For the rocket motor studied in this study, further development in reliability can be achieved by decreasing the variation in the propellant properties, because the sensitivity analyses show that the main source of the variation in rocket motor performance is due to the variation in burn rate of the propellant.

VIII. Conclusions

By using RSM and MCS, the reliability of a solid rocket motor was estimated and a new design point was proposed to increase the reliability. It was shown that the response surface method can be used to estimate performance functions of the motor with enough accuracy. By using response surfaces, reliability of the rocket motor was assessed at three different operating temperatures: +60, +20, and −35°C. The previous estimations of reliability at these temperatures were 0.988561, 0.998884 and 0.973227, respectively. These results show that the reliability of the rocket motor decreases at the operating temperature limits of −35 and 60°C.

The simulation results show that the most possible failure of the rocket motor is exceeding the maximum acceleration limit at high temperatures and the inability of the rocket motor to provide required total impulse at low temperatures. By considering the failure modes included in this study, a new design point was proposed to increase the reliability. This new design point was obtained by changing the grain geometry dimensions and nozzle dimensions. The improved reliability values are 0.999952 at +60°C, 0.999760 at −35°C, and larger than 0.999999 at 20°C.

It is a known fact that increasing the reliability at later stages of the design is difficult and costly. However, it is shown in this study that even with small changes in the dimensions of the designed motor, the reliability of a rocket motor can easily be increased in early design phases.

Table 6 Expected rocket motor performance for the new design point

	Old values		Expected values	
Performance parameter	Mean at −35°C	Mean at +60°C	Mean at −35°C	Mean at +60°C
Total impulse, N · s	7597	7683.7	7692.8	7797.8
Maximum acceleration, *g*	72.83	90.74	67.96	85.03
Arming acceleration duration, s	1.371	1.100	1.484	1.187
Launch acceleration, *g*	43.14	56.44	43.02	55.87
Maximum pressure, bar	97.8	121.2	89.4	114.2

Table 7 Monte Carlo simulation results for the new design point

	Expected values after optimization		Simulation result on new design point	
Performance parameter	Mean at −35°C	Mean at +60°C	Mean at −35°C	Mean at +60°C
Total impulse, N · s	7692.8	7797.8	7697.6	7802.701
Maximum acceleration, *g*	67.96	85.03	67.96	84.893
Arming acceleration duration, s	1.484	1.187	1.488	1.19
Launch acceleration, *g*	43.02	55.87	43.23	55.92
Maximum pressure, bar	89.4	114.2	86.6	120.5

Table 8 Reliability of the rocket motor for the new design point

	Estimated probability of failure before optimization			Estimated probability of failure after optimization		
Limiting response	60°C	20°C	−35°C	60°C	20°C	−35°C
Total impulse	0.7×10^{-4}	0.001109	0.026693	$<10^{-6}$	$<10^{-6}$	0.97×10^{-4}
Maximum acceleration	0.011369	7×10^{-6}	$<10^{-6}$	0.48×10^{-5}	$<10^{-6}$	$<10^{-6}$
Arming acceleration	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$
Launch acceleration	$<10^{-6}$	$<10^{-6}$	0.000211	$<10^{-6}$	$<10^{-6}$	0.000154
Total probability of failure	0.011439	0.001116	0.026773	0.48×10^{-5}	$<10^{-6}$	0.00024
Reliability	0.988561	0.998884	0.973227	0.9999952	>0.999999	0.99976

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